Remarks on transient photon production in heavy ion collisions

Eduardo Fraga[(1)], François Gelis[(2)], Dominique Schiff[(3)]

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1. Intituto de Física
Universidade Federal do Rio de Janeiro
C.P. 68528, Rio de Janeiro, RJ 21941-972, Brazil

2. Service de Physique Théorique
CEA/DSM/Saclay, Orme des merisiers
91191 Gif-sur-Yvette cedex, France

3. Laboratoire de Physique Théorique
Bât. 210, Université Paris XI
91405 Orsay cedex, France

Abstract

In this note, we discuss the derivation of a formula that has been used in the literature in order to compute the number of photons emitted by a hot or dense system during a finite time. Our derivation is based on a variation of the standard operator-based S-matrix approach. The shortcomings of this formula are then emphasized, which leads to a negative conclusion concerning the possibility of using it to predict transient effects for the photon rate.

1 Introduction

Electromagnetic radiation (photons and lepton pairs) has long been thought to be a good probe of the early stages of heavy ion collisions. Indeed, the production rates of these particles increase very rapidly with temperature and therefore are dominated by the early times. In addition, photons and leptons are interacting very weakly with quarks and gluons, which means that final state interactions can in general be neglected.

The photon and dilepton production rates have been evaluated completely at leading order in the strong coupling α_s for a quark-gluon plasma in *local thermal*

equilibrium. These rates have been evaluated at 1-loop[1] in [3, 4, 5], at 2-loop in [6, 7, 8, 9], and the Landau-Pomeranchuk-Migdal [10, 11, 12] corrections have been resummed in [13, 14, 15, 16, 17, 18, 19]. Up-to-date reviews of the situation regarding photon and dilepton production in equilibrium can be found in [20, 21, 22, 23].

The situation is in a far less advanced state when the system is not in local thermal equilibrium. Some attempts have been made in order to take into account in the calculation the effect of a departure from chemical equilibrium. This is done by introducing *fugacities* in the quark and gluon distributions. The fugacity dependent rates have been evaluated at 1-loop in [24], at 2-loop in [25], and generalized to the infinite series of diagrams that contribute to the LPM effect in [23]. These calculations are based on a minimal extension of the equilibrium formalism, that merely consists in replacing the equilibrium statistical distributions by non-equilibrium distributions. As a consequence, they may suffer from the so-called pinch singularities. It can however be shown that the corrections due to the "pinch terms" are negligible if the process under study is fast compared to the relaxation time [26].

In recent years, a new "real-time" approach has been proposed in order to compute out-of-equilibrium – namely time-dependent – effects for photon production in a dense equilibrated quark-gluon system, which originate in its finite life-time [27, 28]. The result was that there are important transient effects that make the yield much larger than what would have been expected by simply multiplying the equilibrium rates by the corresponding amount of time.

This unexpectedly large photon yield, combined with the fact that the energy spectrum is not integrable[2], was the starting point of many discussions regarding the validity of this approach [31, 32, 23]. In this note, we critically discuss the assumptions underlying the formulas used in [27, 28, 29]. To that effect, we propose a derivation of the formula giving the photon yield which is based on the standard canonical formalism. In particular, we show that it can be obtained from an "almost standard" S-matrix approach, the only departure from the standard being that we need to turn on and off the electromagnetic interactions at some finite times t_i and t_f.

Our paper is organized as follows. Section 2 is devoted to a derivation of the photon yield up to the order e^2 of an expansion in the electromagnetic coupling constant. In section 3, we come back to the assumptions made in the derivation of the previous section, and discuss their relevance and validity. Section 4 is devoted to concluding remarks.

[1] The loop counting refer to diagrams of the effective theory one obtains after resumming the hard thermal loops [1, 2].

[2] This appears to have changed in more recent iterations [29, 30] of this work, thanks to a subtraction of the terms responsible for this behavior.

2 Photon production

2.1 General framework

Let us now consider a system of quarks and gluons, and denote by $H_{_{QCD}}$ its Hamiltonian (containing all the QCD interactions). We couple the quarks to the electromagnetic field in order to study photon emission by this system, and denote $H_{\rm e.m.}$ the Hamiltonian of the electromagnetic field, and $H_{q\gamma}$ the term of the Hamiltonian that couples the quarks to the photons. The complete Hamiltonian is therefore:

$$H = H_{_{QCD}} + H_{\rm e.m.} + H_{q\gamma} \; . \tag{1}$$

At the level of the Lagrangian, this reads:

$$\begin{aligned}
&\mathcal{L} = \mathcal{L}_{_{QCD}} + \mathcal{L}_{\rm e.m.} + \mathcal{L}_{q\gamma} \; ,\\
&\mathcal{L}_{_{QCD}} \equiv -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \overline{\psi}(i\partial\!\!\!/_x - gG\!\!\!/(x) - m)\psi(x) \; ,\\
&\mathcal{L}_{\rm e.m.} \equiv -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \; ,\\
&\mathcal{L}_{q\gamma} \equiv -e\overline{\psi}(x) A\!\!\!/(x)\psi(x) \; ,
\end{aligned} \tag{2}$$

where G_μ and $G^a_{\mu\nu}$ are respectively the gluon field and field strength, A_μ and $F_{\mu\nu}$ the photon field and field strength, and ψ the quark field (only one flavor is considered here). g is the strong coupling constant, and e is the quark electric charge. We denote collectively by $\mathcal{L}_{\rm int}$ the sum of all the interaction terms.

The number of photons measured in the system at some late time is given by the following formula:

$$2\omega \frac{dN}{d^3\boldsymbol{x} d^3\boldsymbol{k}} = \frac{1}{V} \sum_{\text{pol. } \lambda} \frac{1}{Z} {\rm Tr}\Big(\rho(t_i) a^{(\lambda)\dagger}_{\rm out}(\boldsymbol{k}) a^{(\lambda)}_{\rm out}(\boldsymbol{k})\Big) \; . \tag{3}$$

V is the volume of the system and $Z \equiv {\rm Tr}(\rho(t_i))$ the partition function. The sum runs over the physical polarization states of the photon. $\rho(t_i)$ is the density operator that defines the initial statistical ensemble. The "in" states and operators of the interaction picture are free, and are defined to coincide with those of the Heisenberg picture at the initial time $t = t_i$. The "out" states and operators are those used in order to perform the measurement. In principle, the measurement should take place after the photons have stopped interacting, i.e. one should count the photons at a time $t \to +\infty$ so that they are asymptotically free photons. Here, for the sake of the argument, we are going to define the "out" states and fields at some finite time t_f. This means that we assume that electromagnetic interactions have been turned off before the time t_f, for this measurement to be meaningful. These "out" states and fields are related to the "in" states and fields by means of the "S-matrix":

$$\big|\alpha_{\rm out}\big\rangle = S^\dagger \big|\alpha_{\rm in}\big\rangle \; ,$$

$$
\begin{aligned}
a^{(\lambda)}_{\rm out}(\boldsymbol{k}) &= S^\dagger a^{(\lambda)}_{\rm in}(\boldsymbol{k}) S\;,\\
a^{(\lambda)\dagger}_{\rm out}(\boldsymbol{k}) &= S^\dagger a^{(\lambda)\dagger}_{\rm in}(\boldsymbol{k}) S\;,
\end{aligned}
\tag{4}
$$

with an S matrix given in terms of the interaction as[3]:

$$
\begin{aligned}
S &= U(t_f,t_i) \equiv \mathcal{P}\exp i\int_{t_i}^{t_f} d^4x \mathcal{L}_{\rm int}(\phi_{\rm in}(x))\;,\\
S^\dagger &= U(t_i,t_f) \equiv \mathcal{P}\exp i\int_{t_f}^{t_i} d^4x \mathcal{L}_{\rm int}(\phi_{\rm in}(x))\;.
\end{aligned}
\tag{5}
$$

$\mathcal{P}$ denotes the path-ordering. Expressing the "out" creation and annihilation operators in terms of their "in" counterparts, we have:

$$
2\omega\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}} = \frac{1}{V}\sum_{\text{pol. }\lambda}\frac{1}{Z}{\rm Tr}\,\Big(\rho(t_i)S^\dagger a^{(\lambda)\dagger}_{\rm in}(\boldsymbol{k})a^{(\lambda)}_{\rm in}(\boldsymbol{k})S\Big)\;.
\tag{6}
$$

The next step required in order to bring this expression to an easily calculable form is to rewrite the creation and annihilation operators in terms of the corresponding fields[4]:

$$
\begin{aligned}
a^{(\lambda)\dagger}_{\rm in}(\boldsymbol{k}) &= -i\varepsilon^{(\lambda)*}_\mu(\boldsymbol{k})\int d^3\boldsymbol{x} e^{-ik\cdot x}\overleftrightarrow{\partial}_{x_0} A^\mu_{\rm in}(x)\;,\\
a^{(\lambda)}_{\rm in}(\boldsymbol{k}) &= i\varepsilon^{(\lambda)}_\mu(\boldsymbol{k})\int d^3\boldsymbol{x} e^{ik\cdot x}\overleftrightarrow{\partial}_{x_0} A^\mu_{\rm in}(x)\;,
\end{aligned}
\tag{7}
$$

where $\varepsilon^{(\lambda)}_\mu(\boldsymbol{k})$ is the polarization vector for a photon of momentum $\boldsymbol{k}$ and polarization λ. The symbol $\overleftrightarrow{\partial}_{x_0}$ is defined as follows:

$$
A(x_0,\boldsymbol{x})\overleftrightarrow{\partial}_{x_0} B(x_0,\boldsymbol{x}) \equiv A(x_0,\boldsymbol{x})\left(\partial_{x_0}B(x_0,\boldsymbol{x})\right) - \left(\partial_{x_0}A(x_0,\boldsymbol{x})\right)B(x_0,\boldsymbol{x})\;.
\tag{8}
$$

Using the property $U(t_1,t_2)U(t_2,t_3)=U(t_1,t_3)$ and the previous relations, we can rewrite Eq. (6) as follows:

$$
\begin{aligned}
2\omega\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}} &= \frac{1}{V}\sum_{\text{pol. }\lambda}\varepsilon^{(\lambda)*}_\mu(\boldsymbol{k})\varepsilon^{(\lambda)}_\nu(\boldsymbol{k})e^{i\omega\cdot(y_0-x_0)}\overleftrightarrow{\partial}_{x_0}\overleftrightarrow{\partial}_{y_0}\\
&\quad\times\frac{1}{Z}{\rm Tr}\,\left[\rho(t_i)U(t_i,t_f)A^\mu_{\rm in}(x_0,\boldsymbol{k})A^\nu_{\rm in}(y_0,-\boldsymbol{k})U(t_f,t_i)\right]\;,
\end{aligned}
\tag{9}
$$

where we have performed a Fourier transform on the spatial dependence of the photon field:

$$
A^\mu_{\rm in}(x_0,\boldsymbol{k}) \equiv \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} A^\mu_{\rm in}(x_0,\boldsymbol{x})\;.
\tag{10}
$$

[3]Note that this is the S-matrix for a system in which the interactions are switched on at the time t_i and switched off at the the time t_f.

[4]The interaction picture field $A^\mu_{\rm in}(x)$ being a free field, these relations indeed give creation and annihilation operators that are time independent. One can therefore choose arbitrarily the value of x_0 when evaluating Eq. (7).

In Eq. (9), one still has the freedom to chose at will the times x_0 and y_0. This freedom can be exploited in order to write:

$$2\omega\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}} = \frac{1}{V}\sum_{\text{pol. }\lambda} \varepsilon_\mu^{(\lambda)*}(\boldsymbol{k})\varepsilon_\nu^{(\lambda)}(\boldsymbol{k})(\partial_{x_0}+i\omega)(\partial_{y_0}-i\omega)$$
$$\times\frac{1}{Z}\mathrm{Tr}\left[\rho(t_i)\mathcal{P}\left(A_{\text{in}}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\text{in}}^{\nu,(+)}(y_0,-\boldsymbol{k})e^{i\int_{\mathcal{C}}\mathcal{L}_{\text{int}}}\right)\right]_{x_0=y_0=t_f}. \tag{11}$$

In this formula, the time path $\mathcal{C}$ goes from t_i to t_f along the real axis, and then back to t_i. Although it could be included in the definition of $\mathcal{C}$, the portion of the contour that goes from t_f to $+\infty$ and back to t_f would not contribute to the expression (because $U(t_f,+\infty)U(+\infty,t_f)=1$). In Eq. (11), the time derivatives act only on the right (and should be taken before the times x_0 and y_0 are set equal to t_f). The subscript $(-)$ (resp. $(+)$) indicates that the corresponding field lives on the lower (resp. upper) branch of the time-path; they are necessary in order to ensure that the two fields remain correctly ordered when acted upon by the path ordering operator.

2.2 Order e^0

At this point, one can perform an expansion in the electromagnetic coupling constant, while conserving strong interactions to all orders. This is motivated by the very different magnitude of the electromagnetic and the strong coupling constants. The term of order 0 in the electric charge e is obtained by replacing the full interaction Lagrangian $\mathcal{L}_{\text{int}}$ by the QCD interactions only (which we denote $\mathcal{L}_{\text{int}}^{QCD}$), which leads to:

$$2\omega\left.\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}}\right|_{e^0} = \frac{1}{V}\sum_{\text{pol. }\lambda} \varepsilon_\mu^{(\lambda)*}(\boldsymbol{k})\varepsilon_\nu^{(\lambda)}(\boldsymbol{k})(\partial_{x_0}+i\omega)(\partial_{y_0}-i\omega)$$
$$\times\left\langle A_{\text{in}}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\text{in}}^{\nu,(+)}(y_0,-\boldsymbol{k})\right\rangle_{x_0=y_0=t_f}, \tag{12}$$

where we denote:

$$\langle\mathcal{A}\rangle \equiv \frac{\mathrm{Tr}\left[\rho(t_i)\mathcal{P}\mathcal{A}\,e^{i\int_{\mathcal{C}}\mathcal{L}_{\text{int}}^{QCD}}\right]}{\mathrm{Tr}\left[\rho(t_i)\right]} \tag{13}$$

the ensemble average of an operator $\mathcal{A}$ at order zero in the electromagnetic coupling (but to all orders in the strong coupling constant). Since this object is evaluated at order 0 in the electromagnetic coupling constant, the correlator that appears in eq. (12) is nothing but the *free* path-ordered photon propagator (the exponential containing the strong interactions drops out because the photons do not couple to quarks at this order[5]). Therefore, we have in the Feynman

[5]Quarks and gluons can only enter in disconnected vacuum-vacuum diagrams that are zero when the time integrations are carried out on both branches of the closed time path $\mathcal{C}$.

gauge:

$$\left\langle A_{\rm in}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\rm in}^{\nu,(+)}(y_0,-\boldsymbol{k})\right\rangle =$$
$$= -g^{\mu\nu}\frac{V}{2\omega}\left[(1+n_\gamma(\omega))e^{-i\omega(x_0-y_0)}+n_\gamma(\omega)e^{i\omega(x_0-y_0)}\right] \; , \qquad (14)$$

where we have used explicitly the fact that x_0 is always posterior to y_0 on the time-path. We denote by $n_\gamma(\omega)$ the statistical distribution of photons present in the initial ensemble described by $\rho(t_i)$. The prefactor V comes from translation invariance: the correlator is proportional to a momentum conservation delta function $(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k})=V$. Applying the operator $(\partial_{x_0}+i\omega)(\partial_{y_0}-i\omega)$ on the previous correlation function, we obtain trivially:

$$\left.\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}}\right|_{e^0} = -\left[\sum_{\text{pol. }\lambda}\varepsilon_\mu^{(\lambda)*}(\boldsymbol{k})\varepsilon^{(\lambda)\mu}(\boldsymbol{k})\right] n_\gamma(\omega) = 2n_\gamma(\omega) \; . \qquad (15)$$

Naturally, this result was expected: at order $\mathcal{O}(e^0)$, the number of photons in the system is given by the initial photon distribution function multiplied by the number of physical degrees of polarization. This is zero if we assume that the system does not contain any photons initially.

2.3 Order e^2

Expanding $\exp -ie\int\overline{\psi}A\!\!\!/\psi$ to order e^1 produces a 3-photon correlator, which vanishes by Furry's theorem. Therefore, the next non zero contribution can occur only at the order e^2. Expanding the exponential containing the electromagnetic interactions up to this order, we obtain[6]:

$$2\omega \left.\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}}\right|_{e^2} = -\frac{e^2}{2V}\sum_{\text{pol. }\lambda}\varepsilon_\mu^{(\lambda)*}(\boldsymbol{k})\varepsilon_\nu^{(\lambda)}(\boldsymbol{k})(\partial_{x_0}+i\omega)(\partial_{y_0}-i\omega)$$
$$\times\left\langle A_{\rm in}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\rm in}^{\nu,(+)}(y_0,-\boldsymbol{k})L_2\right\rangle \; , \qquad (16)$$

with the shorthand:

$$L_2 \equiv \int_{\mathcal{C}} du_0\, dv_0 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{d^3\boldsymbol{q}}{(2\pi)^3} A_{\rm in}^\rho(u_0,\boldsymbol{p})A_{\rm in}^\sigma(v_0,\boldsymbol{q})J_{{\rm in},\rho}(u_0,-\boldsymbol{p})J_{{\rm in},\sigma}(v_0,-\boldsymbol{q}) \; . \qquad (17)$$

$J_{{\rm in},\rho}=\overline{\psi}_{\rm in}\gamma_\rho\psi_{\rm in}$ is the electromagnetic vector current. The correlator that appears in this formula can now be expanded using Wick's theorem. Naturally, the fermionic currents can only be paired among themselves. One of the pairings corresponds to a term that contains a disconnected vacuum-vacuum factor: such

[6] Implicitly, this is the correct e^2 expansion only if the initial density matrix does not depend on the coupling constant e, i.e. if there are no electromagnetic interactions in the system at the initial time.

a vacuum-vacuum diagram is zero when evaluated on the closed time path $\mathcal{C}$. We are left with:

$$
\begin{aligned}
&\big\langle A_{\rm in}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\rm in}^{\nu,(+)}(y_0,-\boldsymbol{k})L_2\big\rangle\\
&=\int_{\mathcal{C}} du_0\,dv_0\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{d^3\boldsymbol{q}}{(2\pi)^3}\big\langle J_{{\rm in},\rho}(u_0,-\boldsymbol{p})J_{{\rm in},\sigma}(v_0,-\boldsymbol{q})\big\rangle\\
&\qquad\times\Big\{\big\langle A_{\rm in}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\rm in}^{\rho}(u_0,\boldsymbol{p})\big\rangle\big\langle A_{\rm in}^{\nu,(+)}(y_0,-\boldsymbol{k})A_{\rm in}^{\sigma}(v_0,\boldsymbol{q})\big\rangle\\
&\qquad+\big\langle A_{\rm in}^{\mu,(-)}(x_0,\boldsymbol{k})A_{\rm in}^{\sigma}(v_0,\boldsymbol{q})\big\rangle\big\langle A_{\rm in}^{\nu,(+)}(y_0,-\boldsymbol{k})A_{\rm in}^{\rho}(u_0,\boldsymbol{p})\big\rangle\Big\}\;. \qquad (18)
\end{aligned}
$$

The 2-photon correlator is:

$$
\big\langle A_{\rm in}^{\mu}(x_0,\boldsymbol{k})A_{\rm in}^{\nu}(y_0,\boldsymbol{k}')\big\rangle=-(2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')g^{\mu\nu}G(x_0,y_0;\boldsymbol{k})\;, \qquad (19)
$$

where we denote

$$
\begin{aligned}
G(x_0,y_0;\boldsymbol{k})\equiv\frac{1}{2\omega}\big[&(\theta_c(x_0-y_0)+n_\gamma(\omega))e^{-i\omega(x_0-y_0)}\\
&+(\theta_c(y_0-x_0)+n_\gamma(\omega))e^{-i\omega(y_0-x_0)}\big] \qquad (20)
\end{aligned}
$$

with $\theta_c(x_0-y_0)$ the generalization of the step function to the time path $\mathcal{C}$. This formula is a generalization of eq. (14) to the case where the times x_0 and y_0 can lie anywhere on the time path. The current-current correlator can be written in terms of the photon polarization tensor as follows:

$$
\big\langle J_{{\rm in},\rho}(u_0,-\boldsymbol{p})J_{{\rm in},\sigma}(v_0,-\boldsymbol{q})\big\rangle=(2\pi)^3\delta(\boldsymbol{p}+\boldsymbol{q})\Pi_{\rho\sigma}(u_0,v_0;-\boldsymbol{p})\;. \qquad (21)
$$

Acting on eq. (18) with the operator $(\partial_{x_0}+i\omega)(\partial_{y_0}-i\omega)$, we obtain easily:

$$
\begin{aligned}
2\omega\,\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}}\Big|_{e^2}&=-\frac{e^2}{2}\sum_{\text{pol. }\lambda}\varepsilon_\mu^{(\lambda)*}(\boldsymbol{k})\varepsilon_\nu^{(\lambda)}(\boldsymbol{k})\int_{\mathcal{C}}du_0\,dv_0\,e^{i\omega(v_0-u_0)}\\
&\times[n_\gamma(\omega)+\theta_c(u_0-t_f^-)][n_\gamma(\omega)+\theta_c(t_f^+-v_0)]\Pi^{\mu\nu}(u_0,v_0;\boldsymbol{k})\,. \qquad (22)
\end{aligned}
$$

where the superscript $+$ or $-$ on the time t_f indicates on which branch of the contour $\mathcal{C}$ the corresponding time must be considered.

At this point, we can break down the contour integrations in domains where the θ functions have a constant value. Using the property:

$$
\Pi^{\mu\nu}_{++}(u_0,v_0;\boldsymbol{k})+\Pi^{\mu\nu}_{--}(u_0,v_0;\boldsymbol{k})=\Pi^{\mu\nu}_{+-}(u_0,v_0;\boldsymbol{k})+\Pi^{\mu\nu}_{-+}(u_0,v_0;\boldsymbol{k}) \qquad (23)
$$

which is valid even out-of-equilibrium, we obtain:

$$
\begin{aligned}
2\omega\,\frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}}\Big|_{e^2}&=-\frac{e^2}{2}\sum_{\text{pol. }\lambda}\varepsilon_\mu^{(\lambda)*}(\boldsymbol{k})\varepsilon_\nu^{(\lambda)}(\boldsymbol{k})\int_{t_i}^{t_f}du_0\,dv_0\,e^{i\omega(v_0-u_0)}\\
&\times[n_\gamma(\omega)\Pi^{\mu\nu}_{+-}(u_0,v_0;\boldsymbol{k})-(1+n_\gamma(\omega))\Pi^{\mu\nu}_{-+}(u_0,v_0;\boldsymbol{k})]\,. \qquad (24)
\end{aligned}
$$

Performing the sum over the photon polarizations, and combining the order e^0 and the order e^2 results, we have the following photon distribution at time t_f:

$$2\omega \frac{dN}{d^3\boldsymbol{x}d^3\boldsymbol{k}} = 2n_\gamma(\omega) + \frac{e^2}{2}\int\limits_{t_i}^{t_f} du_0 dv_0\, e^{i\omega(v_0-u_0)} \times [n_\gamma(\omega)\Pi^\mu_{\mu+-}(u_0,v_0;\boldsymbol{k}) - (1+n_\gamma(\omega))\Pi^\mu_{\mu-+}(u_0,v_0;\boldsymbol{k})] + \mathcal{O}(e^4)\,. \qquad (25)$$

This formula can be applied to several particular cases, that we discuss in the following sections.

3 Photons, quarks and gluons in thermal equilibrium

A first situation to consider is that of an initial ensemble which corresponds to quarks, gluons, *and photons* in thermal equilibrium at a given temperature T. This means that the initial distributions of these particles are given by the Fermi-Dirac and Bose-Einstein functions at temperature T. Moreover, the initial density operator is $\rho(t_i) = \exp(-H/T)$ where H is the full Hamiltonian of the system, *including all the interactions (strong as well as electromagnetic)*. In this case[7], the photon polarization tensor obeys the KMS identity, which reads:

$$n_\gamma(\omega)\Pi^{\mu\nu}_{+-}(u_0,v_0;\boldsymbol{k}) = (1+n_\gamma(\omega))\Pi^{\mu\nu}_{-+}(u_0,v_0;\boldsymbol{k})\,. \qquad (26)$$

This relation implies that the e^2 term in eq. (25) vanishes. One could in fact check that all the higher order terms (in e^2) in the photon distribution would also vanish thanks to the KMS identity. Naturally, this result is expected: if the initial ensemble is an equilibrium ensemble, the populations of particles do not change over time. It is also natural that in the calculation this property appears as a consequence of KMS: indeed, KMS is the manifestation of thermal equilibrium at the level of the Green's functions. In practice, the situation considered in this section is only of academic interest, because in applications such as heavy ion collisions the photons are not in equilibrium with the strongly interacting particles.

4 Photon-free initial state

More interesting is the situation of a system which does not contain any real photons initially, i.e. for which $n_\gamma(\omega) = 0$. In this case, the photon population

[7]When $\rho(t_i) = \exp(-H/T)$, there is an explicit dependence of the initial density matrix on the coupling constant e. It is known that this dependence is taken care of without changing any formula, simply by appending a vertical branch to the time path [33], going from t_i to $t_i - i/T$. Moreover, it has been shown that the contribution of this vertical branch can be taken into account if one enforces the KMS condition at each intermediate step of the calculation [33, 34, 35].

at time t_f is given by:

$$2\omega \frac{dN}{d^3\boldsymbol{x} d^3\boldsymbol{k}} = -\frac{e^2}{2} \int\limits_{t_i}^{t_f} du_0\, dv_0\, e^{i\omega(v_0-u_0)} \Pi^{\mu}_{\mu-+}(u_0, v_0; \boldsymbol{k}) \;. \tag{27}$$

If the initial density matrix describing the distribution of quarks and gluons is such that $\Pi^{\mu}_{\mu-+}(u_0, v_0; \boldsymbol{k})$ is invariant under time translation, we can introduce the Fourier transform of the photon polarization tensor:

$$\Pi^{\mu}_{\mu-+}(u_0, v_0; \boldsymbol{k}) \equiv \int\limits_{-\infty}^{+\infty} \frac{dE}{2\pi} e^{iE(u_0-v_0)} \Pi^{\mu}_{\mu-+}(E, \boldsymbol{k}) \;, \tag{28}$$

and we can write:

$$2\omega \frac{dN}{d^3\boldsymbol{x} d^3\boldsymbol{k}} = -e^2 \int\limits_{-\infty}^{+\infty} \frac{dE}{2\pi} \frac{1-\cos((E-\omega)(t_f-t_i))}{(E-\omega)^2} \Pi^{\mu}_{\mu-+}(E, \boldsymbol{k}) \;. \tag{29}$$

If one takes the derivative with respect to t_f, one obtains the number of photons produced per unit time and per unit phase-space at the time t_f:

$$2\omega \frac{dN}{dt d^3\boldsymbol{x} d^3\boldsymbol{k}} = -e^2 \int\limits_{-\infty}^{+\infty} \frac{dE}{2\pi} \frac{\sin((E-\omega)(t_f-t_i))}{E-\omega} \Pi^{\mu}_{\mu-+}(E, \boldsymbol{k}) \;. \tag{30}$$

This formula is equivalent to the formula obtained by Boyanovsky et al. However, we have derived it here within the framework of an S-matrix formulation. This was not the standard S-matrix approach though, as some extra hypothesis and some extensions have been used. They are discussed in the next section.

5 Discussion

A first consequence of eq. (30) is that it gives back the usual formula for the photon production rate at equilibrium if one takes to infinity the time t_f at which the measurement is performed (which amounts to turn off adiabatically the electromagnetic interactions only at asymptotic times). Indeed, one has the following property:

$$\lim_{t_f\to+\infty} \frac{\sin((E-\omega)(t_f-t_i))}{E-\omega} = \pi\delta(E-\omega) \;, \tag{31}$$

which implies:

$$2\omega \frac{dN}{dt d^3\boldsymbol{x} d^3\boldsymbol{k}} = -\frac{e^2}{2} \Pi^{\mu}_{\mu-+}(\omega, \boldsymbol{k}) \;, \tag{32}$$

i.e. we recover in this limit the usual relation between the photon production rate and the on-shell photon polarization tensor. In particular, there is no contribution to the rate at zeroth order in α_s.

At this point, the main question is whether the finite t_f generalization of this formula makes sense as a photon production rate, as invoked in [29]. In order to discuss this possibility, it is useful to recall and discuss here the hypothesis that have been used in order to derive eq. (30).

- One may be tempted to interpret the number operator $a^\dagger_{\rm out}a_{\rm out}$ we have defined in the third of eqs. (4) as the number operator at the time t_f for photons still interacting with the system, but there is no warranty that this definition of the number of photons agrees with the number of photons as measured in a detector, precisely because they are not asymptotically free states.

  The only possibility to argue safely that this operator indeed counts observable photons is to assume that the system does not undergo electromagnetic interactions after the time t_f. Another way to state this is to say that the object S which appears in eq. (4) is the standard S-matrix connecting free "out" and "in" states only if there are no interactions after t_f.

  In any case, it is clearly unphysical to keep t_f finite: either we are trying to measure non asymptotically free photons, or we have to turn off the interactions at a finite time t_f.

- A similar problem arises at the initial time. The derivation we have used for eq. (30) assumed that there is no dependence on e in the initial density operator $\rho(t_i)$, which is possible only if there are no electromagnetic interactions in the initial state. In particular, imposing $n_\gamma = 0$ in the initial state is also forbidding electromagnetic interactions before t_i (because a system of interacting quarks and gluons that undergo electromagnetic interactions will necessarily contain photons as well).

- It is also known that eq. (30) is plagued by very serious pathologies that appear as ultraviolet divergences. Firstly, the r.h.s. of eq. (30) turns out to be infinite at any fixed photon energy ω due to some unphysical vacuum contributions, i.e. processes where a photon is produced without any particle in the initial state. Secondly, the remaining terms, even if they give a finite photon production rate, lead to an energy dependence of this rate which is too hard for being integrable: one would conclude based on this formula that the total energy radiated as photons per unit time by a finite volume is infinite, which clearly violates energy conservation.

  It was claimed in [29] that the vacuum terms could be discarded simply by subtracting to the r.h.s. of eq. (30) the same formula evaluated in the vacuum. This indeed has the desired effect, but is a totally *ad hoc* prescription because nowhere in the derivation of the formula appears this subtraction term. Or said differently, since eq. (30) is a direct consequence

of the definition of eq. (3), whatever is wrong with the final formula signals a problem either with this definition or with the model.

Similarly, the authors of [29] suggested that the divergence that appears in the total radiated energy can be subtracted by multiplying the creation and annihilation operators used in the definition of the number of photons by some wave function renormalization constants. However, no such constants appear in the derivation: the operators $a_{\rm out}, a^\dagger_{\rm out}$ can be related to their "in" counterparts directly by means of the S-matrix.

6 Conclusions

In this paper, we have obtained a new simple derivation of a formula recently proposed in order to calculate the number of photons produced by a system during a finite time. The purpose of this derivation is to be sufficiently transparent in order to exhibit all the underlying hypothesis. In particular, in order to obtain this formula, one would have to impose that the electromagnetic interactions be turned off before the initial time t_i and after some final time t_f. This is clearly unphysical and not surprisingly leads to serious pathologies. Our conclusion is that eq. (30) does not make sense when keeping a finite $t_f - t_i$ time interval. The transient effects associated to a finite lifetime of the hot and dense system, which could enhance the photon production as suggested in [29], cannot be evaluated that way.

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